Numerical Techniques

Our goal here is compare several numerical techniques. We will work with functions we can integrate to allow us to check our accuracy. The functions whose area we wish to estimate are the following:

1) $f(x) = x\sin(x) + 1$ from $a = 0$ to $b = \pi$

2) $f(x) = 8x - 7x^2 + \frac{7}{3}x^3 = \frac{x^4}{4}$ from $a = 0$ to $b = 5$

We want to compare four techniques:

1) Left Hand Endpoint, LEFT(n)
2) Midpoint, MID(n)
3) Trapazoid, TRAP(n)
4) Simpsons, SIMP(n)

OUR QUESTION

What is the minimal value of n necessary in order for TECHNIQUE(n) to be within 1 % of the real value.

STEP 1.

Find the real value $\int_a^b fx)\mathtt{d}x$.

Step 2.

Find 1% of that and use this to determine the range values that your numerical result must make it into.

Step 3 (LEFT).

Calculate LEFT(n) on $f(x)$ until your result is in the desired range (determined in STEP 2). Find the smallest such n (or nearly so).

Repeat Step 3 for MID, TRAP, and SIMP.

Fill in the answer sheet on the next page.

Useful Formulas

LEFT(n) $\quad A = \sum_{k=0}^{n-1} \Delta x \cdot f\left(\Delta x \cdot k\right)$

MID(n) $\quad A = \sum_{k=0}^{n-1} \Delta x \cdot f\left(\Delta x \cdot k + \frac{\Delta x}{2}\right)$

TRAP(n) $\quad A = \frac{\Delta x}{3}\left(f(a) + \left[\sum_{k=1}^{n-1} \Delta x \cdot f\left(\Delta x \cdot k\right)\right] + f(b)\right)$

SIMP(n) $\quad A = \frac{2 \cdot MID + TRAP}{3}$

$$f(x) = x\sin(x) + 1$$

TRUE VALUE $\int_0^{\pi} x\sin(x) + 1\mathtt{d}x =$ _____

1 % RANGE _____

	n	Numerical Value
$LEFT(n)$		
$MID(n)$		
$TRAP(n)$		
$SIMP(n)$		

$$f(x) = 8x - 7x^2 + \tfrac{7}{3}x^3 - \tfrac{x^4}{4}$$

TRUE VALUE $\int_0^{5} 8x - 7x^2 + \frac{7}{3}x^3 - \frac{x^4}{4}\mathtt{d}x =$ _____

1 % RANGE _____

	n	Numerical Value
$LEFT(n)$		
$MID(n)$		
$TRAP(n)$		
$SIMP(n)$		