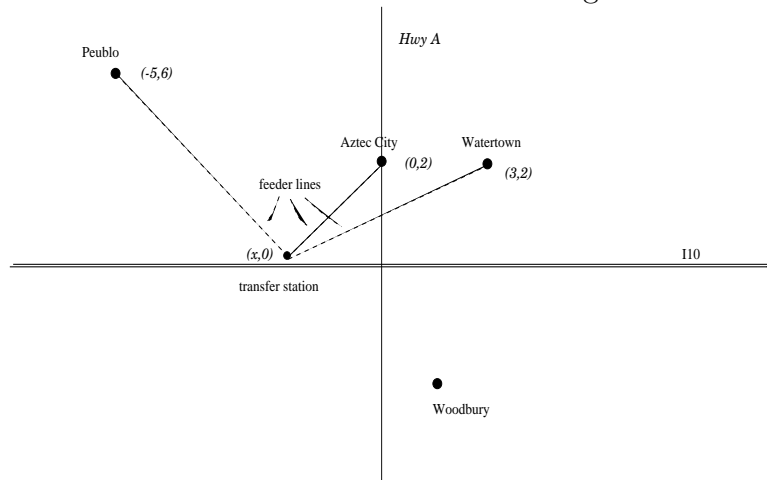


Transmission Lines

Part I. SouthWest Power Company is building a new power transfer station to supply electricity to the three towns of Pueblo, Aztec City and Watertown. Three separate feeder lines will be built which will run straight from the transfer station to each town, one feeder line for each town. SWP Co requires the transfer station to be directly next to the primary trunk lines (BIG power lines) which run parallel to Interstate 10. Our job is to figure out where the station should be in order to minimize the length of the feeder lines.



Highway A intersects I-10 and we will regard I-10 as the x -axis. We will regard Highway A as the y axis where distance is measured in miles. With this coordinate system Pueblo is at the point $(-5, 6)$, Aztec City is at $(0, 2)$ and Watertown is at $(3, 2)$. We let $(x, 0)$ denote the position of the transfer station. The distance from Pueblo to the transfer station is given by

$$d_P = \sqrt{(x - (-5))^2 + 6^2}.$$

The distance from Aztec City to the transfer station is given by

$$d_A = \sqrt{(x - 0)^2 + 2^2}$$

and the distance from Watertown to the transfer station is given by

$$d_W = \sqrt{(x - 3)^2 + 2^2}.$$

Since the feeder line which goes to Pueblo goes straight there from the transfer station the length of the feeder line is given by d_P . For example, if we put the transfer station one mile

east of the intersection of Highway A and I-10, it would be located at $(-1, 0)$ and the length of the feeder line connecting to Pueblo would be

$$d_P = \sqrt{(1 - (-5))^2 + 6^2} = \sqrt{(1 + 5)^2 + 6^2} = \sqrt{36 + 36} = \sqrt{72}.$$

In order to connect all three town to the feeder stations, the total length of feeder line needed is $D = d_P + d_A + d_W$. Written as a function of the feeder station position $(x, 0)$ we have

$$D(x) = \sqrt{(x - (-5))^2 + 6^2} + \sqrt{(x - 0)^2 + 2^2} + \sqrt{(x - 3)^2 + 2^2}$$

And we want to minimize $D(x)$!

So we differentiate $D(x)$ with respect x getting $D'(x)$ and we want to set $D'(x) = 0$ and solve for x . That will tell us where to put the transfer station. Ans: $x = \underline{\hspace{2cm}}$

Hint: The Derive **Solve** feature can be very useful. This feature (accessible in the pull-down menus) can help you locate where a function is zero. Let's go through an example with the function $f(x) = 2x^2 + 3x + 1$.

We wish to find the solution to the equation $2x^2 + 3x + 1 = 0$. To do so, enter the function into Derive,

$$\boxed{f(x):=2x^2+3x+1}$$

$$f(x) := 2 \cdot x^2 + 3 \cdot x + 1$$

Then, with the function highlighted (as though you were going to graph it), click on the *Solve* button on the menu at the top of the screen. A window will open, revealing the options **Expression** or **System**. Choose **Expression** (click on it).

Another window will open, containing four boxes with options and three buttons across the bottom. The left box will be labelled *Solution variable* which should contain an x . (If it doesn't, put an x in it.) The next box will be labelled *Solution method* in which you want to choose *Numerically*. In the box labelled *Solution domain* you want to choose *Real*. (You can ignore the box labelled *Solution bounds*.) Finally, click on the button labelled **Solve** (it should be the center button).

You should see

$$x = -0.5 \vee x = 1$$

The symbol \vee is the mathematical symbol for the word *or*. So what Derive is telling us is that the solution to the equation $2x^2 + 3x + 1 = 0$ is $x = -0.5$ or $x = 1$. Use the *Solve* feature to find where $D'(x) = 0$. (Of course, in our application, we are only interested in the positive answers.)

Part II. What if SWP Co. wants to connect Woodbury to the transfer station, as well. Woodbury is located at $(1, -3)$. Now, where would we want to locate the transfer station in order to minimize the total length of the feeder lines? Ans: $x =$ _____