Newton's Method

In this project we want to practice Newton's Method of estimating zeros. Let us use Newton's method to estimate $\sqrt{7}$. Since $a = \sqrt{7}$ makes $a^2 - 7 = 0$ we will try to estimate $f(x) = x^2 - 7 = 0$. So

$$f'(x) = 2x.$$

Thus, we have

$$x_1 = x_0 - \frac{x_0^2 - 7}{2x_0}$$

To get Derive to compute this we write something like the following.

`x1(x0):=x0–(x0 ^ 2–7)/(2*x0)`

Give Derive a place to begin.

`x0:=1`

Now enter

`x1(x0)`

We see

#☐ $x1(x0)$

#☐ 4

as an answer on the screen. Ah!, so $x1 = 4$. Now, we would like to do that again to calculate $x2$.

We want to go through the same computation with $x1$ and $x2$ replacing $x0$ and $x1$. We could rewrite $x1 = x0 - \dfrac{x0^2 - 7}{2x0}$ replacing $x1$ with $x2$ and replacing $x0$ with $x1$ to get $x2 = x1 - \dfrac{x1^2 - 7}{2x1}$.

But it is easier just to call $x0 = 4$ and redo the calculation using $x1 = x0 - \dfrac{x0^2 - 7}{2x0}$. We enter

`x0:=4` and

`x1(x0)`

getting the answers

#☐ $x1(x0)$

#☐ $\dfrac{23}{8}$.

So $x2 = 23/8$. We can get a decimal approximation using the $\approx$ button to get

#☐ 2.875

By repeating this we continue to compute $x0, x1, x2, x3, x4, ...$

Thus we can fill in the data.

1

$f(x) =$	x0	x1	x2	x3	x4	x5
$x^2 - 7$	1	4	$\frac{23}{8}$			

Alternatively, we could fill in the data in a decimal format.

1)

$f(x) =$	x0	x1	x2	x3	x4	x5
$x^2 - 7$	1	4	2.875			

Since this is a numerical technique, let us continue to work with the decimal format. What happens if we change our initial point?

2)

$f(x) =$	x0	x1	x2	x3	x4	x5
$x^2 - 7$	-1					

What about?

3)

$f(x) =$	x0	x1	x2	x3	x4	x5
$x^2 - 7$	100					

4) Refer to problem 1. Graph $f(x)$ together with the tangent lines that correspond to the values of $x = x0$, $x = x1$, $x = x2$, and $x = x3$. Locate the values of $x0$, $x1$, and $x2$ on the x-axis of that graph.

5) Consider the equation $x = \sin(x)$. This does not have an answer that one can derive algebraically. Can you think of a way to use Newton's method to get a good estimate of the answer? Demonstrate how to do this. Include an explanation of what you are doing.