Linearization

A linearization is a model of a function (say f) that is designed to be a good model near some number (say a). What kind of model, you ask? The model is a straight line. We take f and a, and try to find the straight line $l(x)$ which acts like $f(x)$ for x near a. In this project we want to practice making linearizations. Let us begin by making a linearization of the function $f(x) = \sqrt{x}$ at $a = 9$.

A) Find the equation for the line which is tangent to the graph of the function $f(x) = \sqrt{x}$ above the value $a = 9$.

`Tangent Line:`____

Note: we write $a = 9$ to avoid a confusing proliferation of x s. Make sure you understand how $l(x)$ is constructed.

B) Graph $f(x)$ and $l(x)$ on the same graph.
Label f, l and the value on the x-axis, $a = 9$.

Did it look like what you expected? Check: Are $f(9)$ and $l(9)$ equal?

C) Demonstrate (with a small table) for values of x near $a = 9$, that $f(x) \approx l(x)$. For example,

x	$f(x)$	$l(x)$	$\lvert f(x) - l(x) \rvert$
9	3	3	0
9.5			
8.5			
9.25			
8.75			

D) How does an estimate of $\sqrt{7}$ by this method compare to what you got using Newton's method?

i) $l(7) =$______________________

ii) Newton's method's estimate ______________________

2) Repeat steps A,B, and C for $a = 16$ (with $f(x) = \sqrt{x}$).

A) Find the equation for the line which is tangent to the graph of the function $f(x) = \sqrt{x}$ above the value $a = 16$.

`Tangent Line:`____

B) Graph $f(x)$ and $l(x)$ on the same graph.
Label f, l and the value on the x-axis, $a = 16$.

Did it look like what you expected? Check: Are $f(a)$ and $l(a)$ equal?

C) Demonstrate (with a small table) for values of x near $a = 9$, that $f(x) \approx l(x)$. For example,

x	$f(x)$	$l(x)$	$\lvert f(x) - l(x)\rvert$
16	4	4	0

3) Repeat steps A,B, and C with $f(x) = \log(x)$ for $a = 10$.

A) Find the equation for the line which is tangent to the graph of the function $f(x)$ above the value a.

`Tangent Line:`____

B) Graph $f(x)$ and $l(x)$ on the same graph.
Label f, l and the value on the x-axis, a.

Did it look like what you expected? Check: Are $f(a)$ and $l(a)$ equal?

C) Demonstrate (with a small table) for values of x near a, that $f(x) \approx l(x)$. For example,

x	$f(x)$	$l(x)$	$\|f(x) - l(x)\|$
a			0

4) Repeat steps A,B, and C with $f(x) = \sin(x)$ for $a = \pi/2$.

A) Find the equation for the line which is tangent to the graph of the function $f(x)$ above the value a.

`Tangent Line:`____

B) Graph $f(x)$ and $l(x)$ on the same graph.
Label f, l and the value on the x-axis, a.

Did it look like what you expected? Check: Are $f(a)$ and $l(a)$ equal?

C) Demonstrate (with a small table) for values of x near a, that $f(x) \approx l(x)$. For example,

x	$f(x)$	$l(x)$	$\lvert f(x) - l(x) \rvert$
a			0

A Final Question

At work a customer wanders by and notices your homework lying around and happens to glance at number 4). Their eyes light up. They turn and tell you that they have just that sort of issue. They have a program which must compute the values of $\sin(x)$ for various x repeatedly, millions of times over. Calculating $l(x)$ is MUCH faster and easier than calculating $\sin(x)$. It would be worth a nice fat check (say $10,000) if you could find a line $l(x)$ so that any number x within a distance of 0.1 radians of $\pi/2$ had $l(x)$ within 0.05 of $\sin(x)$. Can you do this?