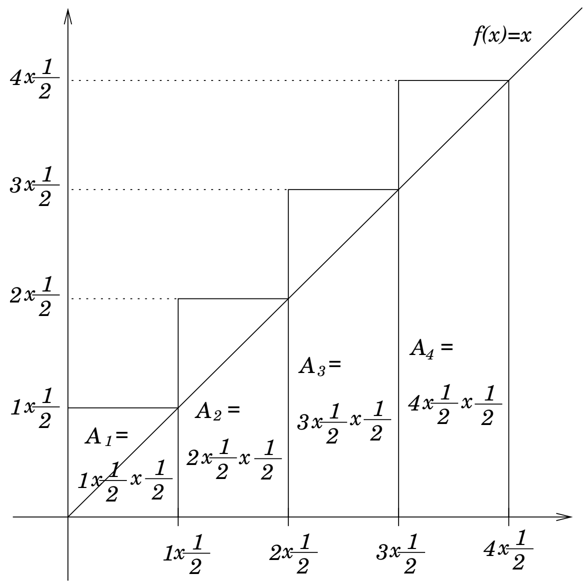
Finding Areas Using Derive

Name______________________

Today we will use Derive to calculate some areas. Our efforts will produce exact answers and will be more-or-less by hand (more-or-less in the sense that derive will be doing the tedious algebra for us, though of course, if we had to we could do the algebra by hand).

We begin with the expression we did in class, $f(x) = x$. Our goal: to find $A_0^2(f)$, the area under the curve $f(x) = x$ between the values of $x = 0$ and $x = 2$. We will begin by getting an expression for the estimate of the area which uses 4 rectangles using the right hand endpoints.



We recall the procedure. First we find the rectangle width,

$$\Delta x = \frac{b-a}{4} = \frac{2-0}{4} = \frac{1}{2}$$

Then we label the numbers on the x-axis.

$$1 \cdot \frac{1}{2}, \quad 2 \cdot \frac{1}{2}, \quad 3 \cdot \frac{1}{2}, \quad 4 \cdot \frac{1}{2}$$

We use these numbers to find the rectangle heights

$$f(1 \cdot \frac{1}{2}), \quad f(2 \cdot \frac{1}{2}), \quad f(3 \cdot \frac{1}{2}), \quad f(4 \cdot \frac{1}{2})$$

Because $f(x) = x$ these are easy to evaluate. For example, $f(1 \cdot \frac{1}{2}) = \frac{1}{2}$ and $f(2 \cdot \frac{1}{2}) = 2 \cdot \frac{1}{2}$ and so forth. So the heights are

$$\frac{1}{2}, \quad 2 \cdot \frac{1}{2}, \quad 3 \cdot \frac{1}{2}, \quad 4 \cdot \frac{1}{2}$$

The area of a rectangle is its height times its width, so the areas are

$$\frac{1}{2} \cdot \frac{1}{2}, \quad 2 \cdot \frac{1}{2} \cdot \frac{1}{2}, \quad 3 \cdot \frac{1}{2} \cdot \frac{1}{2}, \quad 4 \cdot \frac{1}{2} \cdot \frac{1}{2}$$

And we want to add these all up.

$$\frac{1}{2}\cdot\frac{1}{2} \quad + \quad 2\cdot\frac{1}{2}\cdot\frac{1}{2} \quad + \quad 3\cdot\frac{1}{2}\cdot\frac{1}{2} \quad + \quad 4\cdot\frac{1}{2}\cdot\frac{1}{2}$$

We can write this as

$$\text{AREA with 4 rectangles} = \sum_{i=1}^{4} i\,\frac{1}{2}\cdot\frac{1}{2}$$

To get derive to evaluate this enter the following into the command line:

```
SUM(i(1/2)(1/2),i,1,4)
```

Hitting enter and then the [=] button or the [≈] button results in derive producing $\frac{5}{2}$.

Repeat this process using 8 rectangles.

$\Delta x =$ ____________

AREA with 8 rectangles = $\sum$ ________________________

Derive's answer, AREA with 8 rectangles= ____________

Repeat this process using 16 rectangles.

$\Delta x =$ ____________

AREA with 16 rectangles = $\sum$ ________________________

Derive's answer, AREA with 16 rectangles= ____________

Repeat this process using n rectangles.

$\Delta x =$ ____________

AREA with n rectangles = $\sum$ ________________________

Derive's answer, AREA with n rectangles= ________________________

Now, you will probably want derive to expand this, unless you want to do it yourself (it's not hard). At any rate, use the [Simplify] menu button, followed by the [Expand] button (twice). Derive's simplified answer, AREA with n rectangles= ________________________

So in the limit as n gets large the area is ____________

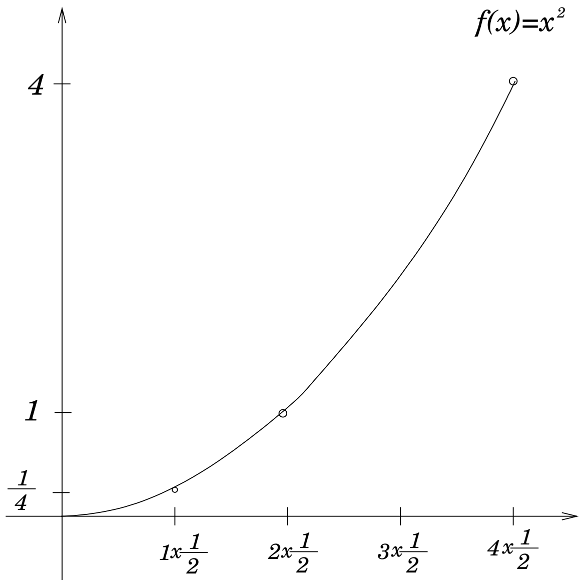
Now, lets repeat this procedure with the function $f(x) = x^2$. Our goal: to find $A_0^2(f)$. We begin with 4 rectangles. As before

$$\Delta x = \frac{1}{2}$$

We label the numbers on the x-axis.

$$1 \cdot \frac{1}{2}, \quad 2 \cdot \frac{1}{2}, \quad 3 \cdot \frac{1}{2}, \quad 4 \cdot \frac{1}{2}$$

Draw in the rectangles in the sketch below



The heights of the rectangles are found with

$$f(1 \cdot \frac{1}{2}), \quad f(2 \cdot \frac{1}{2}), \quad f(3 \cdot \frac{1}{2}), \quad f(4 \cdot \frac{1}{2})$$

Because $f(x) = x^2$ these are also easy to evaluate. For example, $f(1 \cdot \frac{1}{2}) = (\frac{1}{2})^2$ and $f(2 \cdot \frac{1}{2}) = (2 \cdot \frac{1}{2})^2$ and so forth. So the heights are

$$(\frac{1}{2})^2, \quad (2 \cdot \frac{1}{2})^2, \quad (3 \cdot \frac{1}{2})^2, \quad (4 \cdot \frac{1}{2})^2$$

The area of a rectangle is its height times its width, so the areas are

$$(\frac{1}{2})^2 \cdot \frac{1}{2}, \quad (2 \cdot \frac{1}{2})^2 \cdot \frac{1}{2}, \quad (3 \cdot \frac{1}{2})^2 \cdot \frac{1}{2}, \quad (4 \cdot \frac{1}{2})^2 \cdot \frac{1}{2}$$

Sketch in the rectangles in the picture, along with their heights and areas.

And we want to add these all up.

$$(\frac{1}{2})^2 \cdot \frac{1}{2} \quad + \quad (2 \cdot \frac{1}{2})^2 \cdot \frac{1}{2} \quad + \quad (3 \cdot \frac{1}{2})^2 \cdot \frac{1}{2} \quad + \quad (4 \cdot \frac{1}{2})^2 \cdot \frac{1}{2}$$

We can write this as

$$\text{AREA with 4 rectangles} = \sum_{i=1}^{4} (i \ \frac{1}{2})^2 \cdot \frac{1}{2}$$

To get derive to evaluate this enter the following into the command line:

```
SUM((i(1/2)) ^2(1/2),i,1,4)
```

Hitting enter and then the [=] button or the [≈] button results in derive producing $\frac{15}{4}$.

Repeat this process using 8 rectangles.

$\Delta x =$ ______

AREA with 8 rectangles = $\sum$ ____________

Derive's answer, AREA with 8 rectangles= ______

Repeat this process using 16 rectangles.

$\Delta x =$ ______

AREA with 16 rectangles = $\sum$ ____________

Derive's answer, AREA with 16 rectangles= ______

Repeat this process using n rectangles.

$\Delta x =$ ______

AREA with n rectangles = $\sum$ ____________

Derive's answer, AREA with n rectangles= ____________

Derive's simplified answer, AREA with n rectangles=

____

So in the limit as n gets large the area is ______

Using $f(x) = x^4$ repeat the entire process, starting with 4 rectangles.

Using 4 rectangles.

$\Delta x =$ __________

AREA with 4 rectangles = $\sum$ ____________________

Derive's answer, AREA with 4 rectangles= __________

On another piece of paper the sketch the scenario with 4 rectangles.

Using 8 rectangles.

$\Delta x =$ __________

AREA with 8 rectangles = $\sum$ ____________________

Derive's answer, AREA with 8 rectangles= __________

Using 16 rectangles.

$\Delta x =$ __________

AREA with 16 rectangles = $\sum$ ____________________

Derive's answer, AREA with 16 rectangles= __________

Using n rectangles.

$\Delta x =$ __________

AREA with n rectangles = $\sum$ ____________________

Derive's answer, AREA with n rectangles= ____________________

Derive's simplified answer, AREA with n rectangles= ____________________

So in the limit as n gets large the area is __________

Using $f(x) = 3x^3 + x^2 + x + 1$ repeat the entire process, starting with 4 rectangles.

Using 4 rectangles.
$\Delta x =$ ________

AREA with 4 rectangles = $\sum$ ____________________

Derive's answer, AREA with 4 rectangles= ________

On another piece of paper the sketch the scenario with 4 rectangles.

Using 8 rectangles.
$\Delta x =$ ________

AREA with 8 rectangles = $\sum$ ____________________

Derive's answer, AREA with 8 rectangles= ________

Using 16 rectangles.
$\Delta x =$ ________

AREA with 16 rectangles = $\sum$ ____________________

Derive's answer, AREA with 16 rectangles= ________

Using n rectangles.
$\Delta x =$ ________

AREA with n rectangles = $\sum$ ____________________

Derive's answer, AREA with n rectangles= ____________________

Derive's simplified answer, AREA with n rectangles= ____________________

So in the limit as n gets large the area is ________

Check your answers.