Introduction to the Concept of the Limit (Part II).

Our goal in this lab is to understand the idea of a limit of a function using sequences as a tool.

In Part I you gained an understanding of the limit of a sequence of numbers. Here you will use sequences of numbers to learn about the limit of a function.
As you know, each of the following functions does have a limit.

$$\lim_{x\to 2} x^2 + 4x - 5 \qquad \lim_{x\to 1} \frac{x^2+6x-7}{x^3-1} \qquad \lim_{x\to 0} x\sin(\frac{1}{x})$$

We would like to verify this using sequences. First we need some sequences that have 2 as a limit. Consider this sequence

$$\left\{2\frac{1}{2},\ 2\frac{1}{4},\ 2\frac{1}{8},\ 2\frac{1}{2},\ \cdots\right\}$$

We could express it using a formula $a(n) = 2 + \frac{1}{2^n}$. We can check that $\lim_{n\to\infty} a(n) = 2$.

We use $a(n)$ to explore $\lim_{x\to 2} f(x)$ where $f(x) = x^2 + 4x - 5$.
Plug $a(n)$ **in** $f(x)$ to form the function $f(a(n))$.
$F(a(n))$ is a sequence of numbers, sort of like $a(n)$ but more complex. Using the techniques from the last lab, find the following.

$$\lim_{n\to\infty} f(a(n)).$$

Make a graph that supports your claim for this limit.

Use a different sequence that approaches 2 in the limit, say

$$\left\{2-\frac{1}{2},\ 2+\frac{1}{3},\ 2-\frac{1}{4},\cdots\right\}$$

which we can express with the formula $b(n) = 2 + \frac{(-1)^n}{n}$.

Find the following.

$$\lim_{n\to\infty} f(b(n)).$$

Make a graph that supports your claim for this limit. Is the limit the same that you got using $a(n)$?

Make up a third sequence $c(n)$ that approaches 2. Use it to find

$$\lim_{n\to\infty} f(c(n)).$$

Make a graph that supports your claim for this limit. Is the limit the same that you got using $a(n)$ and $b(n)$?

This is how one uses sequence to gather evidence for the value of $\lim_{x\to 2} x^2 + 4x - 5$. Collect these graphs together, write a few lines what you found as limits using each of the three sequences. Write a line that says what you conclude the $\lim_{x\to 2} x^2 + 4x - 5$ is. You have done Part IIA.

For Part IIB and Part IIC you want to do the same thing for the other two limits $\lim_{x\to 1} \frac{x^2+6x-7}{x^3-1}$ and $\lim_{x\to 0} x\sin(\frac{1}{x})$. Of course, for these two limits you will need to use sequences that approach 1 and 0, respectively.

Each of the following functions does not have a limit at a particular point.

$$\lim_{x\to 1} f(x) \quad \text{where } f(x) = \begin{cases} 1 & \text{for } x \le 1 \\ -1 & \text{for } x > 1 \end{cases}$$

and

$$\lim_{x\to 0} \sin(\frac{1}{x})$$

Now, we want to use sequences to find evidence that the limits don't exist. To get a working version of $f(x)$ into Derive use the following.

f(x):=CHI(0,x,1) - CHI(1,x,2)

Try graphing $f(x)$ to make sure it does what you want it to.

We need a sequence that approaches 1, like

$$\left\{1-\frac{1}{2}, 1+\frac{1}{3}, 1-\frac{1}{4}, \cdots\right\}$$

which we can express with the formula $a(n) = 1+\frac{(-1)^n}{n}$. Graph the sequence $f(a(n))$. Does it converge to anything? If it doesn't, then you are done. In order to show

$$\lim_{x\to 1} f(x)$$

does **not** exist all you need to do is find *one* sequence $a(n)$ which approaches 1, for which $f(a(n))$ does not converge. Just one. Have you done this? If so write a brief statement saying so, attach it to your graph (which provides the evidence) and you are done with Part IID.

For Part IIE, do a similar thing for

$$\lim_{x\to 0} \sin(\frac{1}{x}).$$

Can you find a sequence $b(n)$ so that the sequence $\sin(\frac{1}{b(n)})$ looks *exactly* like the one you found in Part IID?

Finally, for Part IIF answer the following question:

QIIF: Does the following limit exist?

$$\lim_{x\to 0} x\ln(x).$$

If so, use sequences to provide evidence for that convergence. If not, use sequences to provide evidence for that.