Derive Differentiation Lab

Calculus 181 Name____________________

In this project you practice going through the motions of differentiating polynomials and rational functions using the definition of the derivative. You do this as an exercise to help cement the definition into your head. The exercise is tolerable because you have Derive available to help you perform the algebra (naturally, you don't need Derive, it just speeds up the process).

Suppose you want to differentiate the function $f(x) = 4x^3 + 6x^2 + 2x + 7$. You know the definition of the derivative:

$$f'(x) = \lim_{c \to x} \frac{f(c) - f(x)}{c - x} \tag{1}$$

So you plug the above function $f(x)$ into equation 1 to get the rather messy looking

$$f'(x) = \lim_{c \to x} \frac{4c^3 + 6c^2 + 2c + 7 - (4x^3 + 6x^2 + 2x + 7)}{c - x} \tag{2}$$

At this point a rather miraculous thing occurs - something sometimes called the factor theorem wanders into your world. The factor theorem guarantees that

$$4c^3 + 6c^2 + 2c + 7 - (4x^3 + 6x^2 + 2x + 7)$$

can be written as a product $(c - x)$ times some polynomial $q(x)$, so that the fraction in Equation 2 cancels nicely.

(Do you recall what the factor theorem says? It's not covered in every precalculus class. It says, for a polynomial $P(x)$, $P(c) = 0$ if and only if $x - c$ divides $P(x)$ exactly, that is $P(x) = Q(x)(x - c)$ where $Q(x)$ is another polynomial.)

$$\frac{4c^3 + 6c^2 + 2c + 7 - (4x^3 + 6x^2 + 2x + 7)}{c - x} = \frac{(c - x)q(x)}{c - x} = q(x) \tag{3}$$

At this point you may just wonder, so what is this polynomial $q(x)$? Well, you'll just have to do the division

$$c - x \;\overline{\big)\; 4c^3 + 6c^2 + 2c + 7 - (4x^3 + 6x^2 + 2x + 7)}.$$

Ahh, say, need help with that? (This is where you call on Derive.) If you ask it to, Derive will quickly give you the following answer: (Remember the *Expand* option in the *Simplify* menu.)

$$4x^2 + 6x + 4xc + 2 + 6c + 4c^2.$$

Now, your original limit (Equation 2) looks like this:

$$f'(x) = \lim_{c \to x} 4x^2 + 6x + 4xc + 2 + 6c + 4c^2.$$

Because the right side is just a polynomial you can (as you recall) evaluate that limit by just plugging in x for c.

$$f'(x) = 4x^2 + 6x + 4xx + 2 + 6x + 4x^2$$

You simplify.

$$f'(x) = 12x^2 + 12x + 2$$

And you have found your derivative - by hand, from the definition (with a little help from Derive).

This is the process you will go through with a variety of functions using Derive to help you through the algebraic difficulties. Each exercise will consist of the following four steps.

1) You write down the fraction (which you recall is the slope of the secant line) .

$$\frac{f(c) - f(x)}{c - x}$$

2) You simplify the fraction, getting some function $q(x)$.

3) You evaluate the following limit by plugging x in for c in $q(x)$.

$$\lim_{c \to x} q(x) = h(x)$$

4) You write down the result

$$f'(x) = h(x)$$

In the case of your example you would have written

1) $\dfrac{f(c) - f(x)}{c - x} = \underline{\dfrac{4c^3 + 6c^2 + 2c + 7 - (4x^3 + 6x^2 + 2x + 7)}{c - x}}$

2) $= \underline{4x^2 + 6x + 4xc + 2 + 6c + 4c^2}$

3) so $\lim_{c \to x} \dfrac{f(c) - f(x)}{c - x} = \underline{4x^2 + 6x + 4xx + 2 + 6x + 4x^2}$

4) or $f'(x) = \lim_{c \to x} \frac{f(c) - f(x)}{c - x} = \underline{12x^2 + 12x + 2}$

Exercise A: Let $f(x) = 7x^3 + 4x^2 + 2x - 1$ and use the definition as demonstrated above to calculate $f'(x)$.

1) $\dfrac{f(c) - f(x)}{c - x} =$ ______________________________

2) $=$ ______________________________

3) so $\lim\limits_{c \to x} \dfrac{f(c) - f(x)}{c - x} =$ ______________________________

4) or $f'(x) = \lim\limits_{c \to x} \frac{f(c)-f(x)}{c-x} =$ ______________________________ Is this right? Can you CHECK?

Exercise B: Let $f(x) = x^8 + 4x^4 + 2x^2$ and use the definition as demonstrated to calculate $f'(x)$.

1) $\dfrac{f(c) - f(x)}{c - x} =$ ______________________________

2) $=$ ______________________________

3) so $\lim\limits_{c \to x} \dfrac{f(c) - f(x)}{c - x} =$ ______________________________

4) or $f'(x) = \lim\limits_{c \to x} \frac{f(c)-f(x)}{c-x} =$ ______________________________ Is this right? Can you CHECK?

Exercise C: Let $f(x) = \dfrac{1}{x+1}$ and use the definition as demonstrated to calculate $f'(x)$.

1) $\dfrac{f(c) - f(x)}{c - x} =$ ______________________________

2) $=$ ______________________________

3) so $\lim\limits_{c \to x} \dfrac{f(c) - f(x)}{c - x} =$ ______________________________

4) or $f'(x) = \lim\limits_{c \to x} \frac{f(c)-f(x)}{c-x} =$ ______________________________ Is this right? Can you CHECK?

Exercise D: Let $f(x) = \dfrac{1}{3x^2 + 4x + 1}$ and use the definition as demonstrated to calculate $f'(x)$.

1) $\dfrac{f(c) - f(x)}{c - x} =$ ____

2) $=$ ____

3) so $\lim\limits_{c \to x} \dfrac{f(c) - f(x)}{c - x} =$ ____

4) or $f'(x) = \lim\limits_{c \to x} \frac{f(c)-f(x)}{c-x} =$ ____ Is this right? Can you CHECK?

Exercise E: Let $f(x) = \dfrac{x + 1}{x - 1}$ and use the definition as demonstrated to calculate $f'(x)$.

1) $\dfrac{f(c) - f(x)}{c - x} =$ ____

2) $=$ ____

3) so $\lim\limits_{c \to x} \dfrac{f(c) - f(x)}{c - x} =$ ____

4) or $f'(x) = \lim\limits_{c \to x} \frac{f(c)-f(x)}{c-x} =$ ____ Is this right? Can you CHECK?

Exercise F: Let $f(x) = \dfrac{5x^2 - 7}{3x^2 + 4x + 1}$ and use the definition as demonstrated to calculate $f'(x)$.

1) $\dfrac{f(c) - f(x)}{c - x} =$ ____

2) $=$ ____

3) so $\lim\limits_{c \to x} \dfrac{f(c) - f(x)}{c - x} =$ ____

4) or $f'(x) = \lim\limits_{c \to x} \frac{f(c)-f(x)}{c-x} =$ ____ Is this right? Can you CHECK?